Left–Right Symmetry Breaking in NJL Approach

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Abstract

We study left–right symmetric models which contain only fermion and gauge boson fields and no elementary scalars. The Higgs bosons are generated dynamically through a set of gauge– and parity–invariant 4-fermion operators. It is shown that in a model with a composite bi-doublet and two triplet scalars there is no parity breaking at low energies, whereas in the model with two doublets instead of two triplets parity is broken automatically regardless of the choice of the parameters of the model. For phenomenologically allowed values of the right–handed scale a tumbling symmetry breaking mechanism is realized in which parity breaking at a high scale μ_R propagates down and eventually causes the electro–weak symmetry breaking at the scale $\mu_{EW} \sim 100 \; GeV$. The model exhibits a number of low and intermediate mass Higgs bosons with certain relations between their masses. In particular, the components of the $SU(2)_L$ Higgs doublet χ_L are pseudo–Goldstone Bosons of an accidental (approximate) $SU(4)$ symmetry of the Higgs potential and therefore are expected to be relatively light.

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A few years ago a very interesting approach to electro–weak symmetry breaking was put forward, the so called "top condensate" model [1, 2, 3, 4]. In this model the low–energy degrees of freedom are just the usual fermions and gauge bosons, i.e. no fundamental Higgs boson is present. Instead, it is assumed that there is a strong attractive interaction in the quark sector which can lead to the formation of a $t\bar{t}$ bound state playing the role of the Higgs scalar. This interaction is assumed to result from new physics at some high–energy scale Λ, the origin and precise nature of which is not specified. At low energies this new physics would manifest itself through non-renormalizable interactions between the usual fermions and gauge bosons. At energies $E \ll \Lambda$ the lowest dimensional operators are most important, which are just the four–fermion (4-f) operators. Assuming that the heaviest top quark drives the symmetry breaking, one arrives at the practically unique gauge–invariant 4-f operator [1, 2, 3, 4]

$$\mathcal{L}_{4f} = G(\overline{Q}_{Li}t_R)(\overline{t}_R Q_{Li}) , \tag{1}$$

where Q_L is the left–handed doublet of the third generation quarks, G is a dimensionful coupling constant, $G \sim \Lambda^{-2}$, and it is implied that the colour indices are summed over within each bracket.

The four-fermion interaction of eq. (1) can be studied analytically in the large N_c (number of colours) limit in the so–called NJL or fermion bubble approximation[1] [5, 6]. For $G > G_{\text{critical}} = 8\pi^2/N_c\Lambda^2$ the electro–weak symmetry is spontaneously broken, the top quark and the $W^{\pm}$ and Z^0 bosons acquire masses, and a composite Higgs scalar doublet $H \sim \overline{t}_R Q_L$ is formed. To obtain phenomenologically acceptable values for the top quark mass m_t one has to assume that the 4-f coupling constant G is very close to its critical value. It has been shown [3] that this is equivalent to the usual fine–tuning of the Higgs boson mass in the Standard Model. Thus, the gauge hierarchy problem has not been solved in the top–condensate approach[2]. In the fermion bubble approximation one obtains a prediction for m_t which depends logarithmically on the scale of new physics Λ and, in addition, one gets the relation $m_H = 2m_t$ for the Higgs boson mass. For $\Lambda \approx 10^{15}\,GeV$ one finds a value of $m_t \approx 165\,GeV$. However, the renormalization group improved calculations taking into account the loops with propagating composite Higgs scalar and gauge bosons result in significantly higher values of the top quark mass, $m_t = 220 - 240\,GeV$ [3].

Nevertheless, the top condensate approach reproduces correctly the structure of the low–energy effective Lagrangian of the Standard Model and demonstrates how the electro–weak symmetry breaking can result from some high–energy dynamics. It is therefore interesting to study whether a similar approach can work in various extensions of the minimal Standard Model.

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[1]We use the well known abbreviation NJL though the paper of Vaks and Larkin was received and published first.

[2]It has been claimed in [7] that taking into account the loops with composite Higgs scalars results in the automatic cancelation of quadratic divergences and solves the gauge hierarchy problem of the Standard Model in the BHL approach. We do not discuss this possibility here.

In this paper we consider dynamical symmetry breaking in left–right symmetric (LR) models based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [8, 9], following the BHL approach to the Standard Model. Left-right-symmetric models in general are very attractive since they treat left–handed and right–handed fermions symmetrically and explain the parity non-conservation at low energies as a result of spontaneous symmetry breaking. It is usually assumed that symmetry breaking in LR models occurs in two steps: first, $SU(2)_R \times U(1)_{B-L}$ breaks down to $U(1)_Y$ at an energy scale μ_R, and second, the remaining Standard Model gauge group is broken down to $U(1)_{em}$ at the electro–weak scale $\mu_{EW} \sim 100\ GeV$. Obviously this more complicated symmetry breaking pattern requires a richer Higgs sector, and it is interesting to investigate whether the above symmetry breaking scenario can be successfully reproduced in a dynamical model with composite Higgs bosons. As in the BHL approach, we will only consider the usual fermions and gauge bosons of the model as elementary particles, with no fundamental Higgs scalars being present, and in addition introduce a set of relevant 4-f interactions stemming from unspecified new physics at a high energy scale Λ. Here we derive our conclusions in the bubble approximation; more complete results including the renormalization group improved predictions will be reported elsewhere [10].

The Higgs sector of the most popular LR model [9] consists of a bi-doublet $\phi \sim (2, 2, 0)$ and two triplets, $\Delta_L \sim (3, 1, 2)$ and $\Delta_R \sim (1, 3, 2)$, where the quantum numbers with respect to the LR gauge group are shown. Assuming that these scalars are bound states of the usual fermions, the following fermionic content reproduces the correct quantum numbers:

$$\phi_{ij} \sim \alpha(\overline{Q}_{Rj}Q_{Li}) + \beta(\tau_2\overline{Q}_L Q_R \tau_2)_{ij} + \text{leptonic terms} ,$$

$$\vec{\Delta}_L \sim (\Psi_L^T C\tau_2\vec{\tau}\Psi_L), \quad \vec{\Delta}_R \sim (\Psi_R^T C\tau_2\vec{\tau}\Psi_R) . \qquad (2)$$

Here Q_L, Ψ_L (Q_R, Ψ_R) are left–handed (right–handed) doublets of quarks and leptons, respectively; i and j are isospin indices.

In models with Higgs bosons generated by 4-f operators the composite scalars are, roughly speaking, "square roots" of these 4-f operators. One can therefore obtain the above composite Higgs bosons starting from the 4-f operators which are "squares" of the expressions in eq. (2). A convenient way to study models with composite Higgs bosons is the auxiliary field technique, in which one introduces the static auxiliary scalar fields (with appropriate quantum numbers) with Yukawa couplings and mass terms but no kinetic terms and no quartic couplings. Since the modified Lagrangian of the system is quadratic in these auxiliary fields they can always be integrated out in the functional integral [11]. Equivalently, one can use the equations of motion for these fields to express them in terms of the fermionic degrees of freedom. After substituting the resulting expressions into the auxiliary Lagrangian one reproduces the initial 4-f structures.

The static auxiliary fields can acquire gauge–invariant kinetic terms and quartic self–interactions through radiative corrections and become physical propagating scalar fields at

low energies provided that the corresponding gap equations are satisfied [3]. The kinetic terms and mass corrections can be derived from the 2–point Green function, whereas the quartic couplings are given by the 4–point functions. Given the Yukawa couplings of the scalar fields one can readily calculate these functions in the fermion bubble approximation, in which they are given by the corresponding 1-fermion–loop diagrams.

Consider now spontaneous parity breakdown in LR models with composite Higgs bosons. It is usually assumed that, in addition to the gauge symmetry, the Lagrangian of the LR model possesses the discrete parity symmetry under which

$$Q_L \leftrightarrow Q_R, \quad \Psi_L \leftrightarrow \Psi_R, \quad \phi \leftrightarrow \phi^\dagger, \quad \Delta_L \leftrightarrow \Delta_R, \quad W_L \leftrightarrow W_R \ . \tag{3}$$

Even if the Higgs potential of the model is exactly symmetric with respect to the discrete parity transformation, parity can be spontaneously broken by $\langle \Delta_R \rangle > \langle \Delta_L \rangle$ [12]. It is easily seen that this can only occur provided $\lambda_2 > \lambda_1$ where λ_1 and λ_2 are the coefficients of the $[(\Delta_L^\dagger \Delta_L)^2 + (\Delta_R^\dagger \Delta_R)^2]$ and $2(\Delta_L^\dagger \Delta_L)(\Delta_R^\dagger \Delta_R)$ quartic couplings in the Higgs potential. While in the conventional approach λ_1 and λ_2 can be chosen appropriately as free parameters of the model, the scalar mass terms and couplings in the composite Higgs approach are not arbitrary; they are all calculable in terms of the 4-f couplings G_a and the scale of new physics Λ [3]. In particular, in the fermion bubble approximation at one loop level the quartic couplings λ_1 and λ_2 are induced through the Majorana–like Yukawa couplings $f(\Psi_L^T C \tau_2 \vec{\tau} \vec{\Delta}_L \Psi_L + \Psi_R^T C \tau_2 \vec{\tau} \vec{\Delta}_R \Psi_R) + h.c.$, and are given by the diagrams of Fig. 1. It can be seen from Fig. 1b that to induce the λ_2 term one needs the Ψ_L–Ψ_R mixing in the
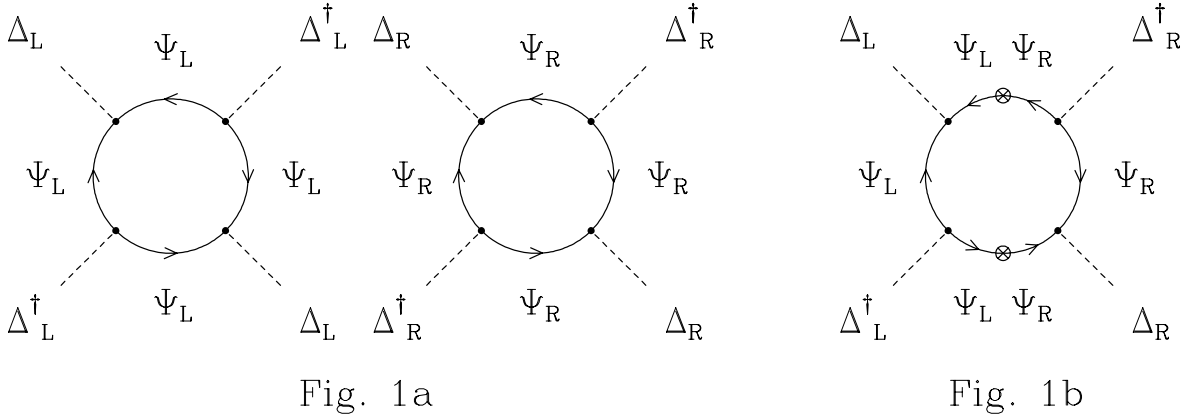

Fig. 1a   Fig. 1b

Figure 1: *Fermion loop diagrams contributing to the quartic couplings λ_1 (Fig. 1a) and λ_2 (Fig. 1b) for Higgs triplets.*

fermion line in the loop, i.e. the lepton Dirac mass term insertions. However, the Dirac mass terms are generated by the VEVs of the bi-doublet ϕ; they are absent at the parity breaking scale which is supposed to be higher than the electro–weak scale. Even if parity and electro–weak symmetry are broken simultaneously (which is hardly a phenomenologically viable scenario), this would not save the situation since the diagram of Fig. 1b is

finite in the limit $\Lambda \to \infty$ whereas the one of Fig. 1a is logarithmically divergent and so the inequality $\lambda_2 > \lambda_1$ cannot be satisfied.

Therefore one is lead to consider a model with a different composite Higgs content. The simplest LR model [8] includes two doublets, $\chi_L \sim (2, 1, -1)$ and $\chi_R \sim (1, 2, -1)$, instead of the triplets Δ_L and Δ_R. As we shall see, the model with composite doublets will automatically result in the correct pattern of the dynamical breakdown of parity.

Since we want the doublet scalars to be composite, we require an additional gauge–singlet fermion as a necessary constituent. We therefore assume that in addition to the usual quark and lepton doublets there is a gauge–singlet fermion

$$S_L \sim (1, \ 1, \ 0) \ . \tag{4}$$

To maintain the discrete parity symmetry one needs a right–handed counterpart of S_L. This can be either another particle, S_R, or the right–handed antiparticle of S_L, $(S_L)^c \equiv C\overline{S}_L^T = S_R^c$. The latter choice is more economical and, as we shall see, leads to the desired symmetry breaking pattern. We therefore assume that under parity operation

$$S_L \leftrightarrow S_R^c \ . \tag{5}$$

With this new singlet and the usual quark and lepton doublets we introduce the following set of gauge–invariant 4-f interactions:

$$\mathcal{L}_{4f} = G_1(\overline{Q}_{Li}Q_{Rj})(\overline{Q}_{Rj}Q_{Li}) + [G_2(\overline{Q}_{Li}Q_{Rj})(\tau_2\overline{Q}_LQ_R\tau_2)_{ij} + h.c.]$$

$$+ G_3(\overline{\Psi}_{Li}\Psi_{Rj})(\overline{\Psi}_{Rj}\Psi_{Li}) + [G_4(\overline{\Psi}_{Li}\Psi_{Rj})(\tau_2\overline{\Psi}_L\Psi_R\tau_2)_{ij} + h.c.]$$

$$+ [G_5(\overline{Q}_{Li}Q_{Rj})(\overline{\Psi}_{Rj}\Psi_{Li}) + h.c.] + [G_6(\overline{Q}_{Li}Q_{Rj})(\tau_2\overline{\Psi}_L\Psi_R\tau_2)_{ij} + h.c.]$$

$$+ G_7[(S_L^TC\Psi_L)(\overline{\Psi}_LC\overline{S}_L^T) + (\overline{S}_L\Psi_R)(\overline{\Psi}_RS_L)] + G_8(S_L^TCS_L)(\overline{S}_LC\overline{S}_L^T) \ . \tag{6}$$

In analogy to the BHL model the G_a are dimensionful 4-f couplings of the order of Λ^{-2} motivated by some new physics at Λ. Notice that the above interactions are not only gauge–invariant, but also (for hermitian G_2, G_4, G_5 and G_6) symmetric with respect to the discrete parity operation (3), (5).

We assume that only the third generation fermions contribute to $\mathcal{L}_{4f}$, i.e., deal with a limit where only the heaviest fermions are massive while all the light fermions are considered to be massless. This seems to be a good starting point from where light fermion masses could, e.g., be generated radiatively. In addition to the bidoublet ϕ of the structure given in eq. (2), the above 4-f couplings, if critical, can give rise to a pair of composite doublets χ_L and χ_R and also to a singlet scalar σ:

$$\chi_L \sim S_L^TC\Psi_L, \quad \chi_R \sim \overline{S}_L\Psi_R = (S_R^c)^TC\Psi_R, \quad \sigma \sim \overline{S}_LC\overline{S}_L^T \ . \tag{7}$$

From eqs. (3) and (5) it follows that under parity we have $\chi_L \leftrightarrow \chi_R$ and $\sigma \leftrightarrow \sigma^\dagger$. Switching to the auxiliary field formalism, the scalars χ_L, χ_R, ϕ and σ have the following bare mass

terms and Yukawa couplings:

$$
\begin{aligned}
L_{aux} &= -M_0^2(\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R) - M_1^2 \operatorname{tr}(\phi^\dagger \phi) - \frac{M_2^2}{2}\operatorname{tr}(\phi^\dagger \widetilde{\phi} + h.c.) - M_3^2 \sigma^\dagger \sigma \\
&\quad - \left[\overline{Q}_L(Y_1\phi + Y_2\widetilde{\phi})Q_R + \overline{\Psi}_L(Y_3\phi + Y_4\widetilde{\phi})\Psi_R + h.c.\right] \\
&\quad - \left[Y_5(\overline{\Psi}_L \chi_L S_R^c + \overline{\Psi}_R \chi_R S_L) + Y_6(S_L^T C S_L)\sigma + h.c.\right] ,
\end{aligned}
\tag{8}
$$

where the field $\widetilde{\phi} \equiv \tau_2 \phi^* \tau_2$ has the same quantum numbers as ϕ: $\widetilde{\phi} \sim (2,\ 2,\ 0)$. By integrating out the auxiliary scalar fields one can reproduce the 4-f structures of eqs. (6) and express the 4-f couplings $G_1, ..., G_8$ in terms of the Yukawa couplings $Y_1, ..., Y_6$ and the mass parameters M_0^2, M_1^2, M_2^2 and M_3^2 (explicit formulas can be found in [10]). In components, the scalar multiplets of the model are

$$
\phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix}, \quad \langle\phi\rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}, \quad \chi_L = \begin{pmatrix} \chi_L^0 \\ \chi_L^- \end{pmatrix}, \quad \chi_R = \begin{pmatrix} \chi_R^0 \\ \chi_R^- \end{pmatrix}.
\tag{9}
$$

Let us now consider parity breaking in the present LR model. In a viable scenario the $SU(2)_R$ symmetry should be broken at the right–handed scale μ_R by $\langle\chi_R^0\rangle = v_R$, and the electro–weak symmetry has to be broken at μ_{EW} by the VEVs of ϕ and possibly of χ_L^0 ($\equiv v_L$). Using the Yukawa couplings of the doublets χ_L and χ_R (see eq. (8)), one can calculate the fermion-loop contributions to the quartic couplings $\lambda_1[(\chi_L^\dagger \chi_L)^2 + (\chi_R^\dagger \chi_R)^2]$ and $2\lambda_2(\chi_L^\dagger \chi_L)(\chi_R^\dagger \chi_R)$ in the effective Higgs potential (Fig. 2a and 2b). The λ_1 and λ_2
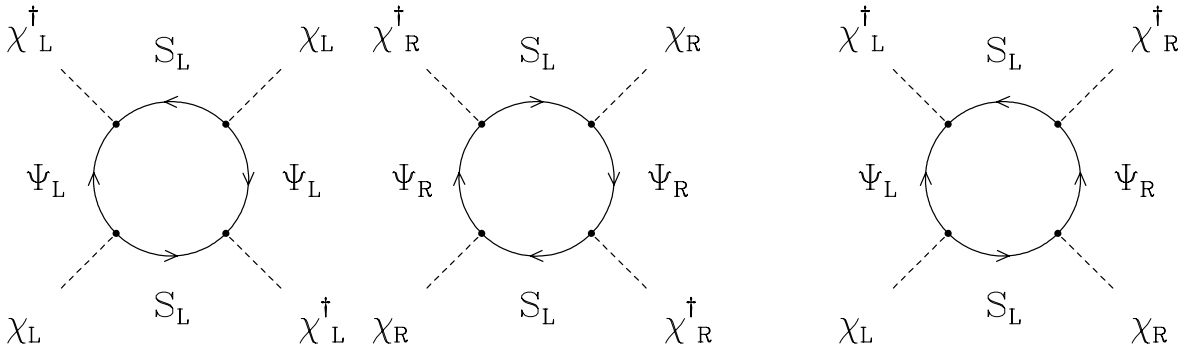

Figure 2: *Fermion loop diagrams contributing to the quartic couplings λ_1 (Fig. 2a) and λ_2 (Fig. 2b) for the Higgs doublets $\chi_{L/R}$.*

terms are now given by similar diagrams. Since the Yukawa couplings of χ_L and χ_R coincide (which is just the consequence of the discrete parity symmetry), Figs. 2a and 2b yield $\lambda_1 = \lambda_2$. Recall that one needs $\lambda_2 > \lambda_1$ to have spontaneous parity breakdown in the LR models. As we shall see, taking into account the gauge boson loop contributions to λ_1 and λ_2 will automatically secure this relation.

Both λ_1 and λ_2 obtain corrections from $U(1)_{B-L}$ gauge boson loops, whereas only λ_1 is corrected by diagrams with W_L^i or W_R^i loops (see Fig. 3). Since all these contributions have
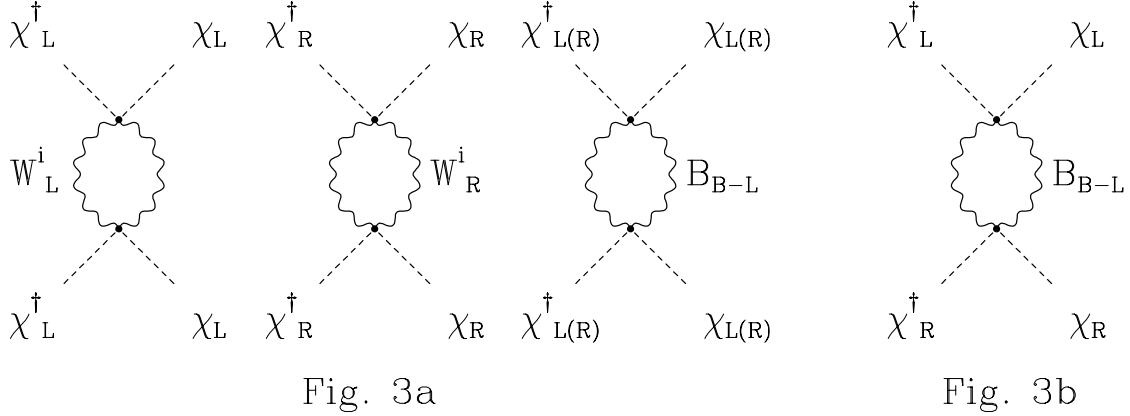

Fig. 3a   Fig. 3b

Figure 3: *Gauge boson loop diagrams contributing to the quartic couplings λ_1 (Fig. 3a) and λ_2 (Fig. 3b) for the Higgs doublets $\chi_{L/R}$ in Landau gauge.*

a relative minus sign compared to the fermion loop ones, one finds $\lambda_2 > \lambda_1$ irrespective of the values of the Yukawa or gauge couplings or any other parameter of the model, provided that the $SU(2)$ gauge coupling $g_2 \neq 0$ [compare the expressions for λ_1 and λ_2 in (20) below]. Thus the condition for spontaneous parity breakdown is automatically satisfied in our model.

We have a very interesting situation here. In a model with composite triplets Δ_L and Δ_R parity is never broken, i.e. the model is not phenomenologically viable. At the same time, in the model with two composite doublets χ_L and χ_R instead of two triplets (which requires introduction of an additional singlet fermion S_L) parity is broken automatically. This means that, unlike in conventional LR models, in the composite Higgs approach *whether or not parity can be spontaneously broken depends on the particle content of the model rather than on the choice of the parameters of the Higgs potential.*

From eq. (8) one can readily find the fermion masses. The masses of the quarks and charged leptons and the Dirac neutrino mass m_D are given by the VEVs of the bi–doublet (we assume all the VEVs to be real):

$$m_t = Y_1\kappa + Y_2\kappa', \qquad m_D = Y_3\kappa + Y_4\kappa',$$

$$m_b = Y_1\kappa' + Y_2\kappa, \qquad m_\tau = Y_3\kappa' + Y_4\kappa. \tag{10}$$

It is well known that LR models with only doublet Higgs scalars usually suffer from the large neutrino mass problem. It turns out that introducing the singlet fermion S_L not only provides the spontaneous parity breaking in our model, but also cures the neutrino mass problem. In fact, as it was first noticed in [13], with an additional singlet neutral

fermion S_L the neutrino mass matrix takes the form (in the basis (ν_L, ν_L^c, S_L))

$$M_\nu = \begin{pmatrix} 0 & m_D & \beta \\ m_D & 0 & M \\ \beta & M & \widetilde{\mu} \end{pmatrix} , \qquad (11)$$

where the entries β, M and $\widetilde{\mu}$ can be read off from eq. (8),

$$\beta = Y_5 v_L , \quad M = Y_5 v_R \quad \widetilde{\mu} = 2 Y_6 \sigma_0 , \qquad (12)$$

with $\sigma_0 \equiv \langle \sigma \rangle$. For $v_R \gg \kappa, \kappa', v_L$ and $v_R \gtrsim \sigma_0$ one obtains two heavy Majorana neutrino mass eigenstates with the masses $\sim M$ and a light Majorana neutrino with the mass $m_\nu \simeq \widetilde{\mu}(m_D^2/M^2) - 2\beta m_D/M$ which vanishes in the limit $M \to \infty$. This is the modified seesaw mechanism which provides the smallness of neutrino mass.

As we mentioned before, radiative corrections in the auxiliary field formalism result in gauge–invariant kinetic terms, quartic interactions and renormalized mass terms for the scalar fields at low energies $E < \Lambda$. The effective low–energy Lagrangian of the system in the bubble approximation can be written as[3]

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + Z_\phi \, \text{tr} \left[(D^\mu \phi)^\dagger (D_\mu \phi) \right] + Z_\chi \left[(D^\mu \chi_L)^\dagger (D_\mu \chi_L) + (D^\mu \chi_R)^\dagger (D_\mu \chi_R) \right]$$
$$+ Z_\sigma (\partial^\mu \sigma)^\dagger (\partial_\mu \sigma) + \mathcal{L}_{Yuk} + V_{\text{eff}} , \qquad (13)$$

where $\mathcal{L}_0$ contains the gauge–invariant kinetic terms of fermions and gauge bosons and $\mathcal{L}_{Yuk}$ is given by the Yukawa–coupling terms in eq. (8). The scalar wave-function renormalization constants are

$$Z_\phi = \frac{1}{16\pi^2} \left[N_c (Y_1^2 + Y_2^2) + Y_3^2 + Y_4^2 \right] \ln \left(\frac{\Lambda^2}{\mu^2} \right) ,$$

$$Z_\chi = \frac{1}{16\pi^2} Y_5^2 \ln \left(\frac{\Lambda^2}{\mu^2} \right) , \quad Z_\sigma = \frac{1}{16\pi^2} 2 Y_6^2 \ln \left(\frac{\Lambda^2}{\mu^2} \right) . \qquad (14)$$

Further, the effective Higgs potential V_{eff} in eq. (13) is given by

$$V_{\text{eff}} = \widetilde{M}_0^2 (\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R) + \widetilde{M}_1^2 \, \text{tr} \, (\phi^\dagger \phi) + \frac{\widetilde{M}_2^2}{2} \, \text{tr} \, (\phi^\dagger \widetilde{\phi} + h.c.) + \widetilde{M}_3^2 \sigma^\dagger \sigma$$

$$+ \lambda_1 [(\chi_L^\dagger \chi_L)^2 + (\chi_R^\dagger \chi_R)^2] + 2\lambda_2 (\chi_L^\dagger \chi_L)(\chi_R^\dagger \chi_R) + \frac{1}{2} \lambda_3 [\chi_L^\dagger (Y_3 \phi + Y_4 \widetilde{\phi}) \chi_R \sigma^\dagger + h.c.]$$

$$+ \lambda_4 [\chi_L^\dagger (Y_3 \phi + Y_4 \widetilde{\phi})(Y_3 \phi^\dagger + Y_4 \widetilde{\phi}^\dagger) \chi_L + \chi_R^\dagger (Y_3 \phi^\dagger + Y_4 \widetilde{\phi}^\dagger)(Y_3 \phi + Y_4 \widetilde{\phi}) \chi_R]$$

$$+ \lambda_5 (\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R) \, \text{tr}(\phi^\dagger \phi) + \lambda_6 (\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R) \sigma^\dagger \sigma$$

$$+ \lambda_7' \, \text{tr}(\phi^\dagger \phi \phi^\dagger \phi) + \frac{1}{3} \lambda_8' \, \text{tr}(\phi^\dagger \widetilde{\phi} \widetilde{\phi}^\dagger \phi) + \frac{1}{12} \lambda_8' [\text{tr}(\phi^\dagger \widetilde{\phi} \phi^\dagger \widetilde{\phi}) + h.c.]$$

$$+ \frac{1}{2} \lambda_9 [\text{tr}(\phi^\dagger \phi \phi^\dagger \widetilde{\phi}) + h.c.] + \lambda_0 [\text{tr}(\phi^\dagger \phi)]^2 + \lambda_{10} (\sigma^\dagger \sigma)^2 . \qquad (15)$$

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[3]For a detailed derivation of $\mathcal{L}_{\text{eff}}$ see [10].

Here we give explicitly only the mass terms and the quartic couplings which we will refer to later, the complete set is given in [10].

$$\widetilde{M}_0^2 \;=\; M_0^2 - \frac{1}{8\pi^2}\left[Y_5^2 - \frac{3}{8}Z_\chi(3g_2^2 + g_1^2)\right](\Lambda^2 - \mu^2) \tag{16}$$

$$\widetilde{M}_1^2 \;=\; M_0^2 - \frac{1}{8\pi^2}\left\{\left[N_c(Y_1^2 + Y_2^2) + (Y_3^2 + Y_4^2)\right] - \frac{9}{4}Z_\phi g_2^2\right\}(\Lambda^2 - \mu^2) \tag{17}$$

$$\widetilde{M}_2^2 \;=\; M_2^2 - \frac{1}{4\pi^2}(N_c Y_1 Y_2 + Y_3 Y_4)(\Lambda^2 - \mu^2) \tag{18}$$

$$\widetilde{M}_3^2 \;=\; M_0^2 - \frac{1}{4\pi^2}Y_6^2\,(\Lambda^2 - \mu^2) \tag{19}$$

$$\lambda_1 \;=\; \frac{1}{16\pi^2}\left[Y_5^4 - \frac{3}{16}(3g_2^4 + 2g_2^2 g_1^2 + g_1^4)Z_\chi^2\right]\ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$\lambda_2 \;=\; \frac{1}{16\pi^2}\left[Y_5^4 - \frac{3}{16}g_1^4 Z_\chi^2\right]\ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$\lambda_0 \;=\; \frac{1}{16\pi^2}\left[-\frac{3}{2}g_2^4 Z_\phi^2\right]\ln\left(\frac{\Lambda^2}{\mu^2}\right)\,, \qquad \lambda_5 \;=\; \frac{1}{16\pi^2}\left[-\frac{9}{8}g_2^4 Z_\phi Z_\chi\right]\ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

$$\lambda_7' \;=\; \frac{1}{16\pi^2}\left[N_c(Y_1^4 + Y_2^4) + (Y_3^4 + Y_4^4)\right]\ln\left(\frac{\Lambda^2}{\mu^2}\right)\,, \qquad \lambda_7 \;=\; \lambda_7' + \lambda_0\,. \tag{20}$$

Here g_2 and g_1 are the $SU(2)$ and $U(1)_{B-L}$ gauge couplings, respectively. The parameters of the above effective Lagrangian depend on the energy scale μ, i.e. they are the running parameters[4]. At $\mu \to \Lambda$ the kinetic terms and quartic couplings of the scalar fields vanish, their mass terms are driven towards their bare values, and one recovers the Lagrangian with auxiliary static scalar fields.

While the bare mass parameters M_i^2 in eq. (8) are positive, the corresponding running quantities $\widetilde{M}_i^2$, given by eqs. (16) – (19), may become negative at low energy scales provided that the corresponding Yukawa couplings are large enough. Those values for which this occurs at $\mu = 0$ we shall call the *critical* Yukawa couplings. For $\widetilde{M}_i^2$ to become negative at some scale $\mu^2 > 0$ the corresponding Yukawa couplings or combinations of them must be above their critical values. If this is to happen at scales $\mu \ll \Lambda$ the Yukawa couplings must be fine-tuned very closely to their critical values to ensure the proper cancelation between the large bare masses of the scalars and the Λ^2 corrections in eqs. (16) – (19). This is equivalent to the usual fine–tuning problem of gauge theories with elementary Higgs scalars [3].

We assume that the scale μ_R at which parity gets spontaneously broken (i.e. χ_R^0

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[4]This bubble–approximation running exactly coincides with the running one would get from 1–loop renormalization group equations keeping only trace terms in the relevant β functions and imposing the compositeness boundary conditions [3]. The results of the renormalization group study with the full 1–loop β functions will be reported in [10].

develops a VEV) is higher than the electro–weak scale $\mu_{EW} \sim 100 \; GeV$, i.e. that $\widetilde{M}_0^2$ changes its sign at a higher scale than $\widetilde{M}_1^2$. This means that $Y_5^2 - (3/8)Z_\chi(3g_2^2 + g_1^2)$ should be bigger than $\tilde{Y}^2 - \frac{9}{4}Z_\phi g_2^2$ [see eqs. (16) and (17)], where $\tilde{Y}^2 \equiv N_c(Y_1^2 + Y_2^2) + (Y_3^2 + Y_4^2)$. The analysis of the vacuum structure in our model [10] shows that if the condition

$$Y_5^2 - \frac{3}{8}Z_\chi(3g_2^2 + g_1^2) > 2\,Y_6^2 \tag{21}$$

is satisfied, either χ_R or χ_L (but not both) acquire a VEV but the σ field does not, whereas for the opposite condition σ acquires a non-zero VEV but not χ_R or χ_L. Clearly the latter situation is phenomenologically unacceptable, but by choosing the 4-f couplings G_7 and G_8 accordingly [10] we can easily satisfy eq. (21).

Let us now discuss the vacuum structure below the electro–weak breaking scale. The non-vanishing VEVs are v_R, κ and κ'. Since $m_t \gg m_b$, it follows from eq. (10) that κ should be much larger than κ' or vice versa provided no significant cancelation between $Y_1\kappa'$ and $Y_2\kappa$ occurs. Without loss of generality one can take $\kappa \gg \kappa'$. To further simplify the discussion, we shall make the frequently used assumption [14] $\kappa' = 0$. The relation $m_t \gg m_b$ then translates into $Y_1 \gg Y_2$. In the conventional approach this assumption does not lead to any contradiction with phenomenology. However, as we shall see, in our case the condition $\kappa' = 0$ cannot be exact.

Consistency of the first–derivative conditions with $\kappa' = 0$ requires $Y_1Y_2 = 0$, $Y_3Y_4 = 0$ and $M_2^2 = 0$ (this gives $\widetilde{M}_2^2 = \lambda_9 = 0$, and as follows from eq. (15), all the terms in the effective potential which are linear in κ' become zero in this limit, as they should). The condition $Y_1Y_2 = 0$ along with $\kappa' = 0$ implies that either $Y_1 = 0$, $m_t = 0$ or $Y_2 = 0$, $m_b = 0$. The first possibility is obviously phenomenologically unacceptable whereas the second one can be considered as a reasonable first approximation; we therefore assume $Y_1 \neq 0$ and $Y_2 = 0$. The situation is less clear for the lepton Yukawa couplings Y_3 and Y_4. Since $m_\tau \ll m_t$ and the Dirac mass m_D of ν_τ is unknown, one can choose either $Y_3 \neq 0$, $Y_4 = 0$ or $Y_3 = 0$, $Y_4 \neq 0$. It turns out that the vacuum stability condition in our model requires $Y_4^2 > Y_3^2$, therefore we choose $Y_3 = 0$ and $Y_4 \neq 0$.

For $\sigma_0 = v_L = \kappa' = Y_2 = Y_3 = 0$ one can easily find expressions for the VEVs of χ_R and ϕ [10]. Approximate expressions in terms of the parity breaking scale μ_R and the electro–weak breaking scale μ_{EW} are

$$v_R^2 \simeq \left(\frac{M_0^2}{\Lambda^2}\right)\frac{\mu_R^2}{2\lambda_1}, \qquad \kappa^2 \simeq \left(\frac{M_0^2}{\Lambda^2}\right)\frac{\mu_{EW}^2}{2\lambda_7} \;, \tag{22}$$

and the ratio of the squared VEVs can be written as

$$\frac{\kappa^2}{v_R^2} \simeq \left(\frac{\lambda_1}{\lambda_7}\right)\frac{\mu_{EW}^2}{\mu_R^2} \sim \frac{\mu_{EW}^2}{\mu_R^2} \simeq \frac{|\lambda_5|}{2\lambda_1} + \frac{\mu_1^2}{\mu_R^2} \;. \tag{23}$$

The parity breaking scale μ_R is the scale where the effective mass term $\widetilde{M}_0^2$ becomes negative for a given Yukawa coupling $Y_5 > (Y_5)_{crit}$ (formally $\mu_R^2 < 0$ for sub–critical Y_5),

while μ_1 is the scale, different from μ_{EW}, where this happens for the mass term $\widetilde{M}_1^2$ and a given $\widetilde{Y}^2$.

Recall now that in conventional LR models with $\mu_{EW} \ll \mu_R \ll \Lambda_{GUT}$ (or Λ_{Planck}) one has to fine-tune two gauge hierarchies: Λ_{GUT}—μ_R and μ_R—μ_{EW}. We have a similar situation here: to achieve $\mu_{EW} \ll \mu_R \ll \Lambda$ one has to fine-tune two Yukawa couplings, Y_5^2 and $\widetilde{Y}^2$. Tuning of Y_5^2 allows for the hierarchy $\mu_R^2 \ll \Lambda^2$; one then needs to adjust $\widetilde{Y}^2$ (or μ_1^2) to achieve $\mu_{EW}^2 \ll \mu_R^2$ through eq. (23).

Since λ_5 only contains relatively small gauge couplings while $Y_5 \sim \mathcal{O}(1)$, we typically have $|\lambda_5|/2\lambda_1 \sim 10^{-2}$. Thus, if there is no significant cancellation between the two terms in (23), one obtains a right–handed scale of the order of a few TeV. Unfortunately, such a low LR scale scenario is not viable. As we shall see below, the squared masses of two Higgs bosons in our model become negative (i.e. the vacuum becomes unstable) unless $v_R \gtrsim 20\ TeV$. This requires some cancellation[5] in eq. (23), and then the right-handed scale $v_R \sim \mu_R$ can in principle lie anywhere between a few tens of TeV and Λ. However, if one prefers "minimal cancellation" in eq. (23), by about two orders of magnitude or so, one would arrive at a value of v_R around $20\ TeV$. In any case it is interesting that the partial cancellation of the two terms in (23) implies $\mu_1^2 < 0$, i.e. that $\widetilde{Y}^2$ must be below its critical value. This means that $\widetilde{M}_1^2$ never becomes negative. In fact it is the $\widetilde{M}_0^2$ term, responsible for parity breakdown, that also drives the VEV of the bi-doublet. It follows from the condition $\partial V_{\text{eff}}/\partial \kappa = 0$ that the effective driving term for κ is $\widetilde{M}_1^2 + \lambda_5 v_R^2$ in our model; it may become negative for large enough v_R^2 even if $\widetilde{M}_1^2$ is positive (remember that $\lambda_5 < 0$). Thus we have a tumbling scenario where the breakdown of parity and $SU(2)_R$ occurring at the scale μ_R causes the breakdown of the electro–weak symmetry at a lower scale μ_{EW}.

To calculate physical observables one should first rescale the Higgs fields so as to absorb the Z factors in eq. (13) into the definitions of the scalar fields and bring their kinetic terms into the canonical form. This amounts to dividing the (mass)^2 terms by the corresponding Z factors, Yukawa couplings by $\sqrt{Z}$ and multiplying the scalar fields and their VEVs by $\sqrt{Z}$. Renormalization factors of the quartic couplings depend on the scalars involved and can be readily read off from the effective potential. We will use hats (ˆ) to denote quantities in the new normalisation.

As we already pointed out, the minimization of the effective Higgs potential gives $\sigma_0 = 0 = v_L$. This means that the entries β and $\widetilde{\mu}$ in the neutrino mass matrix (11) are zero. As a result we have an exactly massless neutrino eigenstate and two heavy Majorana neutrinos with degenerate masses $\sqrt{M^2 + m_D^2}$ and opposite CP–parities which combine to form a heavy Dirac neutrino. Since $m_D \ll M$ the electro–weak eigenstate $\nu_L \equiv \nu_\tau$ is predominantly the massless eigenstate whereas the right–handed neutrino ν_R

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[5]Notice that this does not increase the number of the parameters to be tuned but just shifts the value to which one of them should be adjusted.

and the singlet fermion S_L consists predominantly of the heavy eigenstates. As mentioned before, in the simplified limit $\kappa' = 0$ that we are mainly considering we have $Y_2 = 0 = Y_3$. This yields $m_t = Y_1\kappa$, $m_\tau = Y_4\kappa$ and $m_b = m_D = 0$. Vanishing Dirac neutrino mass m_D implies absence of neutrino mixing, and the heavy neutrino mass is now $M = Y_5 v_R$. From eqs. (10) and (14) and the definition of the renormalized Yukawa couplings one can readily find

$$\hat{\kappa}^2 = (174\,GeV)^2 = N_c m_t^2 \left(1 + \frac{Y_4^2}{N_c Y_1^2}\right) \frac{1}{16\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right) \approx \frac{m_t^2 N_c}{16\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right) \equiv m_t^2 N_c l_0 \,. \quad (24)$$

Here $\hat{\kappa}$ (or $\sqrt{\hat{\kappa}^2 + \hat{\kappa}'^2}$ for $\kappa' \neq 0$) should be identified with the usual electro–weak VEV. This expression coincides with the one derived in the bubble approximation by BHL [3]. Eq. (24) gives the top quark mass in terms of the known electro–weak VEV and the scale of new physics Λ. For example, for $\Lambda = 10^{15}\,GeV$ one finds $m_t \simeq 165\,GeV$. However, this result is limited to the bubble approximation, and the renormalization group improved result for $\kappa' = 0$ turns out to be substantially higher[6] [10]. Notice that $m_t \approx 180\,GeV$, which is the central value of the Fermilab results [15, 16], would mean $l_0 \approx 1/3$. Similar considerations lead to the following relation between the right-handed VEV v_R, the heavy neutrino mass M and the scale Λ:

$$\hat{v_R^2} = M^2 \frac{1}{16\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right) = M^2 \cdot l_0 \,. \quad (25)$$

Note that $\mu \approx m_t$ is understood in eq. (24), whereas $\mu \approx M$ in eq. (25). However, we assume $m_t, M \ll \Lambda$ and $M/m_t \ll \Lambda/M$ throughout this paper, therefore $\ln\frac{\Lambda^2}{m_t^2} \approx \ln\frac{\Lambda^2}{M^2}$, i.e. the logarithmic factor l_0 is universal. From eqs. (24) and (25) one thus finds

$$\frac{\hat{v_R^2}}{M^2} \approx \frac{1}{3}\frac{\hat{\kappa}^2}{m_t^2} \,. \quad (26)$$

The mass of the τ lepton is not predicted in our model since it is only weakly coupled to the bi-doublet; it is given by $m_\tau = (Y_4/Y_1)m_t$ and can be adjusted to a desirable value by choosing the proper magnitude of the ratio Y_4/Y_1, or G_3/G_1.

The composite Higgs bosons in our model include the would-be Goldstone Bosons $G_1^\pm \approx \chi_R^\pm$ (eaten by $W_R^\pm$), $G_2^\pm = \phi_1^\pm$ (eaten by $W_L^\pm$), $G_1^0 = \chi_{Ri}^0$ (eaten by Z_R) and $G_2^0 = \phi_{1i}^0$ (eaten by Z_L). The physical Higgs boson sector of the model contains two CP–even neutral scalars $H_1^0 \approx \chi_{Rr}^0$ and $H_2^0 \approx \phi_{1r}^0$ with the masses

$$M_{H_1^0}^2 \simeq 4M^2 \left[1 - \frac{3}{16}\left(3g^4 + 2g^2 g'^2 + g'^4\right) l_0^2\right] \approx 4M^2 \,, \quad (27)$$

$$M_{H_2^0}^2 \simeq 4m_t^2 \left(1 - \frac{m_\tau^2}{3m_t^2} - \frac{9}{4}g^4 l_0^2\right) \approx 4m_t^2 \,, \quad (28)$$

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[6]The renormalization group improved values of m_t will be viable for appropriate values of $\tan\beta \equiv \kappa/\kappa'$ which in fact is a free parameter in our model depending on the ratio Y_4/Y_3. In the limit $\tan\beta \to \infty$ one obtains too high a top mass, e.g. $m_t = 233\,GeV$ for $\Lambda = 10^{15}\,GeV$, whereas for $\tan\beta = (2.1 - 2.8)$, $\Lambda = 10^{15}\,GeV$ and $\mu_R = 10^7\,GeV$ one finds $m_t = (168 - 192)\,GeV$ [10].

which are directly related to the two steps of symmetry breaking, $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$ and $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$. The mass of the scalar H_2^0, which is the analog of the Standard Model Higgs boson [eq. (28)], essentially coincides with the one obtained in the bubble approximation by BHL [3]. This just reflects the fact that this boson is the $t\bar{t}$ bound state with a mass of $\approx 2m_t$. Analogously, the mass of the heavy CP–even scalar $H_1^0 \approx \chi_{Rr}^0$ is approximately $2M$ since it is a bound state of heavy neutrinos.

Further, there are the charged Higgs bosons $H_3^\pm \approx \phi_2^\pm$ with their neutral CP–even and CP–odd partners $H_{3r}^0 = \phi_{2r}^0$ and $H_{3i}^0 = \phi_{2i}^0$, and finally the χ_L-fields $H_4^\pm = \chi_L^\pm$, $H_{4r}^0 = \chi_{Lr}^0$ and $H_{4i}^0 = \chi_{Li}^0$ with the masses

$$M_{H_3^\pm}^2 \approx \frac{2}{3} M^2 \frac{m_\tau^2}{m_t^2} \, , \tag{29}$$

$$M_{H_{3r}^0}^2 = M_{H_{3i}^0}^2 \approx \frac{2}{3} M^2 \frac{m_\tau^2}{m_t^2} - \frac{1}{2} M_{H_2^0}^2 \, , \tag{30}$$

$$M_{H_4^\pm}^2 = \frac{3}{8} \left(3g_2^4 + 2g_2^2 g_1^2 \right) l_0^2 M^2 + 2m_\tau^2 \tag{31}$$

$$M_{H_{4r}^0}^2 = M_{H_{4i}^0}^2 = \frac{3}{8} \left(3g_2^4 + 2g_2^2 g_1^2 \right) l_0^2 M^2 \, . \tag{32}$$

In conventional LR models only one scalar, which is the analog of the Standard Model Higgs boson, is light (at the electro–weak scale), all the others have their masses of the order of the right–handed scale M [8, 9, 14, 17]. In our case, the masses of those scalars are also proportional to M, but all of them except the mass of H_1^0 have some suppression factors. The mass of the charged scalars $H_3^\pm \approx \phi_2^\pm$ is suppressed by the factor m_τ/m_t and is therefore of the order $10^{-2} M$. The masses of the neutral H_{3r}^0 and H_{3i}^0 are even smaller; they are related to the masses of of the charged $H_3^\pm$ and the Standard Model Higgs H_2^0 by eq. (30). From the vacuum stability condition $M_{H_3^0}^2 > 0$ one thus obtains an upper limit on the Standard Model Higgs boson mass $M_{H_2^0}$ (for a given M) or a lower limit on the right–handed mass M (for a given $M_{H_2^0}$). For example, for $M_{H_2^0} \approx 60 \ GeV$ we find $M \gtrsim 5 \ TeV$. However, since in the top condensate approach the Standard Model Higgs mass is $\sim 2m_t$ (or $\sim m_t$ after the renormalization group improvement), a stronger bound on the right handed gauge symmetry breaking scale of about $20 \ TeV$ results.

The masses of the χ_L scalars [eqs. (31),(32)] vanish in the limit $(\lambda_2 - \lambda_1) \rightarrow 0$ (i.e. $g_2 \rightarrow 0$) and $m_\tau \rightarrow 0$. This fact has a simple interpretation. In the limit $\lambda_2 = \lambda_1$ (which corresponds to the fermion-bubble level) the (χ_L, χ_R) sector of the effective Higgs potential [eq. (15)] depends on χ_L and χ_R only through the combination $(\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R)$. This means that the potential has a global $SU(4)$ symmetry which is larger than the initial $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry. After χ_R^0 gets a non-vanishing VEV v_R, the symmetry is broken down to $SU(3)$, resulting in $15 - 8 = 7$ Goldstone Bosons. Three of them ($\chi_R^\pm$ and $\text{Im} \chi_R^0$) are eaten by the $SU(2)_R$ gauge bosons $W_R^\pm$ and Z_R, and the remaining four ($\chi_L^\pm$, $\text{Re} \chi_L^0$ and $\text{Im} \chi_R^0$) are physical massless Goldstone Bosons. The $SU(4)$

symmetry is broken by the ϕ–dependent terms in the effective potential and by $SU(2)$ gauge interactions. As a result, $\chi_L^\pm$, $\mathrm{Re}\,\chi_L^0$ and $\mathrm{Im}\,\chi_L^0$ acquire small masses and become pseudo–Goldstone Bosons. In fact, the origin of this approximate $SU(4)$ symmetry can be traced back to the 4-f operators of eq. (6). It is an accidental symmetry resulting from the gauge invariance and parity symmetry of the G_7 term. Note that no such symmetry occurs in conventional LR models.

Finally, we would like to comment on the approximation $\kappa' = 0$ which we have used. If we relax this condition, we will obtain non-vanishing masses m_b and m_D (notice that the Yukawa couplings Y_2 and Y_3 will also be non-zero in this case). However, these masses are not predicted in our model and can simply be adjusted to desirable values. The Dirac neutrino mass m_D is unknown and so remains a free parameter; however, it must be smaller than m_τ in our model in order to satisfy the vacuum stability condition $Y_4^2 - Y_3^2 > 0$ [10] which is equivalent to $m_\tau^2 - m_D^2 > 0$. For $\kappa' \ll \kappa$ our predictions for the Higgs boson masses are only slightly modified. As our renormalization–group analysis performed in [10] shows, some interesting results emerge for sizeable values of κ'. The Higgs boson masses and mass eigenstates for the general case $\kappa' \neq 0$ can be found in [10].

In our model we have 9 input parameters (eight 4-f couplings $G_1, ..., G_8$ and the scale of new physics Λ) in terms of which we calculate 16 physical observables (5 fermion masses, 8 Higgs boson masses and 3 VEVs κ, κ' and v_R), so there are $16 - 9 = 7$ predictions. In the simplified case $\kappa' = 0$ that we were mainly considering we have only 5 input parameters since $\kappa' = 0$ requires $G_2 = G_4 = G_5 = 0$, $G_6 = \sqrt{G_1 G_3}$. At the same time we have only 13 non-trivial physical observables since the bottom quark mass and Dirac neutrino mass vanish identically in this case. This yields $13 - 5 = 8$ predictions.

To summarize, this is to our knowledge the first successful attempt to break LR symmetry dynamically. We find a tumbling scenario where the breaking of parity and $SU(2)_R$ eventually drives the breaking of the electro–weak symmetry. The model gives a viable top quark mass value and exhibits a number of low and intermediate scale Higgs bosons. Furthermore it predicts relations between masses of various scalars and between fermion and Higgs boson masses which are in principle testable. If the right–handed scale μ_R is of the order of a few tens of TeV, the neutral CP–even and CP–odd scalars ϕ_{2r}^0 and ϕ_{2i}^0 can be even lighter than the electro–weak Higgs boson. In fact, they can be as light as $\sim 50\ GeV$ and so might be observable at LEP2. Such light ϕ_{2r}^0 and ϕ_{2i}^0 can also provide a positive contribution to $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to hadrons)$ [18] which is necessary to account for the discrepancy between the LEP observations and the Standard Model predictions.

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